

Universal First-Order Reliability Concept Applied to Semistatic Structures

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NOMENCLATURE

e	= uncertainty error
F	= stress, ksi
F'	= statistically derived stress, ksi
K	= probability range factor with sample size
N	= probability range factor
n	= statistical sample size
p	= probability of failure
R	= reliability
SF	= conventional safety factor
Z	= reliability safety index
φ	= disparity coefficient
λ	= stress distribution zone, ksi
μ	= statistical mean, ksi
σ	= standard deviation, ksi
η	= coefficient of variation, σ/μ

Subscripts

A	= applied variable
D	= uncertainty design variable
R	= resistive variable
e	= uncertainty error
T	= test derived variable
o	= midzone stress

TECHNICAL PAPER

UNIVERSAL FIRST-ORDER RELIABILITY CONCEPT APPLIED TO SEMISTATIC STRUCTURES

SUMMARY

Access to space can no longer afford nonoptimum semistatic structural designs with nonuniform and undefined reliability as currently practiced by the conventional deterministic safety-design method. The deterministic pass-fail concept is shown to be genetically flawed and cannot support risk assessments, nor can it design uniformly reliable structures. Stress audits based on safety factor margins alone are incapable of identifying the weakest regions of critical semistatic structures.

The proposed method combines the first-order reliability method with prevailing deterministic design parameters into a reliability criterion to surmount deterministic deficiencies. A universal approach for normalizing applied- and resistive-stress probability distributions is suggested for their adaptation to the simplest and most developed first-order reliability method. A reliability design factor is derived from a reliability criterion to size the structure to a specified reliability in a wide variety of problems, and to verify the reliability criterion response. Reliability selection criteria are briefly discussed. The proposed method is compatible with the current deterministic method, and with the analytical skills, practices, and culture of most structural designers.

INTRODUCTION

Precepts of robust aerostructural designs are to sustain broadly the operational environments with no detrimental deformation, reliably over a given duration, and to achieve it all at least-life cycle cost. There are many generic probability techniques investigated and evolving¹ for designing reliable structures, but the proposed first-order reliability is more compatible with current deterministic computational and verification techniques, and with design practices on semistatic structures. This unique integration of prevailing methods and practices should facilitate the bridging of skills, discipline codes, and user confidence with the simplest and most developed reliability method.

Semistatic structures represent the behavior of over 60 percent of flight hardware. Because of their performance and cost influence on space access, this study was extended over a previous investigation² to craft a more versatile reliability design method through the universal normalization of probability distributions, establishing design criteria to size structures to specified reliabilities in a variety of practical cases, and to incorporate the method in reliability selection criteria development.

But first, a basic understanding of the failure concept and the deterministic method was necessary to develop the prerequisite normal distribution techniques leading to the universal reliability method.

FAILURE CONCEPT

Failure occurs when demand exceeds capability. When applied stresses and material strengths are defined by probability distributions, the probability of failure increases as their tail-overlap area increases, as shown in figure 1. The overlap area suggests the probability that a weak material will encounter an excessively applied stress to cause failure. The probability of failure decreases as the designer-controlled differences of the distribution means increases and the natural distribution shapes decrease.

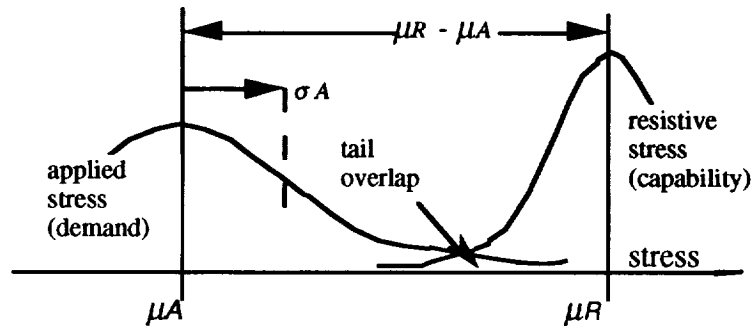


Figure 1. Failure concept.

Distribution shapes are fixed by their natural scatter of data about their means. Shapes are modeled by distribution functions to estimate the probability of a desired value for an assigned range of probability. As shapes become more complex, distribution models become more difficult, and skills and labor to apply them escalate. The normal probability distribution is the easiest, most developed, and best known. It is symmetrical and is simply and completely defined by the mean and standard deviation. It is most compatible with deterministic design parameters.

As in most engineering applications, only data from the worst-case sides of interacting distributions are used and data from the disengaged halves are superfluous. This knowledge and the central-limit theory led to the presumption that structural probability demand and capability distributions may be normalized by constructing a mirror image of the engaged side about its peak frequency value and calculating the standard deviation from the constructed symmetrical distribution. This universal normalizing technique of observed structural data generalized its application to first-order techniques.

DETERMINISTIC METHOD

The deterministic method assumes that the structural safety of a system may be specified by a safety factor defined as the ratio of single-valued minimum resistive and maximum applied stresses,

$$SF = \frac{F_R}{F_A} \quad (1)$$

These single-value parameters are reduced from normal tolerance limits³ developed by their respective stress, loads, and materials disciplines,

$$SF = \frac{\mu_R - K \sigma_R}{\mu_A + N_A \sigma_A}. \quad (2)$$

Resistive stresses are derived from uniaxial tensile-yield and ultimate-stress test data and are characterized into normal tolerance limits with the probability range factor, K , to adjust for sample size for one-sided distributions, as shown in figure 2.

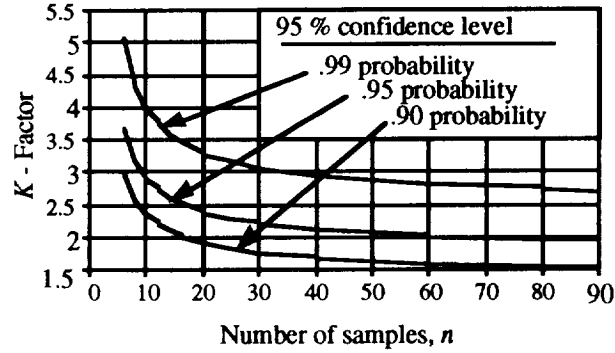


Figure 2. Probability range factor versus sample size.

The applied stress consists of multiaxial, multisource induced-stress components combined into a uniaxial tolerance-limit stress through the Mises criterion,⁴

$$F_A = \left[F_x^2 + F_y^2 + F_z^2 - F_x F_y - F_x F_z - F_y F_z + 3(F_{xy}^2 + F_{xz}^2 + F_{yz}^2) \right]^{\frac{1}{2}}, \quad (3)$$

to be compatible with the interfacing uniaxial-resistive stress.

Three distinct stress zones emerge when constructing statistical variables from equation (2) onto the failure concept shown in figure 3. Each zone is represented by a different structural discipline and each autonomously specifies a probability range through their designer control variables, K , N_A , and SF . The sum of the stress zones is noted to govern the difference of the distribution means and, therefore, their respective structural disciplines independently control a portion of the tail overlap. However, all zones ignore the probability distribution shapes, which was conditioned earlier to size the overlap area and, therefore, this inherent deficiency inhibits the method from predicting structural reliability.

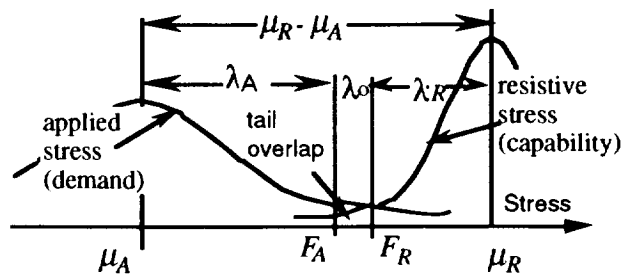


Figure 3. Deterministic failure concept.

Clearly, the end zones govern the difference of the means through the probability range factors of equation (2), and the midzone expands the difference through the safety factor,

$$\lambda_0 = (F_R - F_A) = \left[1 - \frac{1}{SF}\right] F_R.$$

When the safety factor is greater than unity, it effectively increases the applied-stress probability range factor,

$$N_{\text{eff}} = SF \left(\frac{1}{\eta_A} + N_A \right) - \frac{1}{\eta_A}. \quad (4)$$

Though the probability range factor and the safety factor are independently specified, their combined probability range governs the applied-stress distribution tail length and overlap through equation (4). Consequently, a stress audit based on safety factor margins alone is incapable of assessing total relative safety, or identifying the weakest structural region from one component and location to another. Assessing relative safety among different materials gets less credible. A test-verified safety factor margin may exceed specification, but in combination with a small applied-stress probability range factor, may produce a low effective probability range factor or a submarginally stressed region that may not be visible to the analyst.

Having identified and defined probability range factors and relationships to probability distributions, the stress means may be expressed by

$$\mu_R = \frac{F_R}{A} \quad \text{where} \quad A = (1 - K \eta_R), \quad (5)$$

$$\mu_A = \frac{F_A}{B} \quad \text{where} \quad B = (1 + N_A \eta_A). \quad (6)$$

Substituting equations (5) and (6) and the midzone stress into the zones in figure 3, the tail-overlap is governed through the extended difference of the means,

$$\mu_R - \mu_A = \frac{F_R}{A} - \frac{F_R}{B \times SF} + \frac{F_R(SF - 1)}{SF}. \quad (7)$$

PROPOSED FIRST-ORDER RELIABILITY

The classical first-order reliability concept assumes that the applied- and resistive-stress probability density functions are normal and independent. In normalizing the resistive- and applied-stress distributions, as previously discussed, they may be combined to form a third normal expression,⁵

$$Z = \frac{\mu_R - \mu_A}{\sqrt{\sigma_R^2 + \sigma_A^2}}, \quad (8)$$

known as the safety index. The relationship between the safety index and reliability is given by

$$R = P(F_R - F_A > 0) = \phi(Z),$$

where $\phi(z)$ is the standard cumulative distribution, and figure 4 relates equation (8) safety index with reliability.

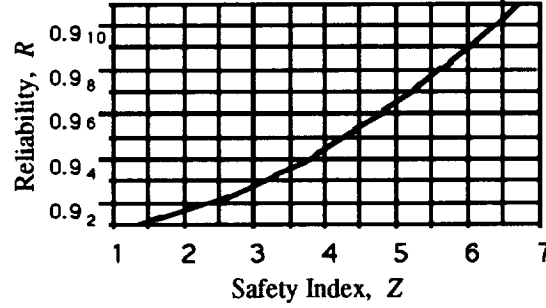


Figure 4. Reliability versus safety index.

Tolerance limits developed in the deterministic method, equations (2) and (7), embody statistical variables and design parameters that may be superimposed on the safety index expression to produce a reliability criterion. A necessary condition is that their combined response tolerance-limits must be statistically derived and explicitly characterized with mean, standard deviation, and probability range factor.

Substituting the extended difference of the distribution means of equation (7) into equation (8) numerator, and combining their related standard deviations,

$$\sigma_R = \mu_R \eta_R = \frac{F_R \eta_R}{A},$$

$$\sigma_A = \mu_A \eta_A = \frac{F_R \eta_A}{B \times SF},$$

into the denominator and simplifying, establishes the proposed reliability criterion,

$$Z = \frac{\phi SF B - A + (\phi SF - 1) B A}{\left[(\phi SF)^2 \eta_R^2 B^2 + \eta_A^2 A^2 \right]^{\frac{1}{2}}}. \quad (9)$$

Solving for the reliability design factor “ ϕSF ” provides the reliability method an equivalent of the deterministic safety factor criterion for calculating the maximum allowed applied stress,

$$\phi SF = \frac{F_R}{F_A}. \quad (10)$$

The reliability design factor accommodates a wide range of applications. The coupled safety factor is the conventional deterministic factor defined by equation (1), and it is specified by NASA as

1.0 on yield and 1.4 on ultimate strengths. It should be cautioned that large safety factors mathematically extend the distribution tails beyond the reality of probable data, and overwhelm other design variables that degenerate the reliability criterion.

The disparity coefficient, ϕ : (a) is selected as unity when calculating the reliability of an existing sized structure; (b) is solved for designing to a specified reliability with design parameters independently selected by interacting discipline; (c) is solved for implementing compensating design uncertainty factors, and for test verification of reliability response; (d) approaches unity as the structural size is optimized with material strength through interfacing discipline control parameters to satisfy a specified reliability.

The reliability method is noted to have established three design criteria and warrant comparison over the deterministic's two. Similar to the deterministic safety factor, the allowable applied-stress is constrained by the reliability design factor criterion, but unlike it, the design factor is derived from the reliability criterion. As in the deterministic method, the structure is sized through the Mises combined tolerance-limit applied stresses and is equated to the maximum allowed stress criterion. But, unlike it, the combined tolerance-limit variables are statistically derived from the Mises criterion and iterated back into the reliability criterion.

APPLICATION

Two basic applications of the method are to size a structure to satisfy a specified reliability, which is an iterative process, and to predict the reliability of an existing sized structure through a direct process.

An approach to sizing a structure is first to estimate the size using the deterministic method to share common design parameters and techniques, and to compare results. Then formulate the multiaxial component stresses into load tolerance-limits with the approximately sized stress-form factors, or into loads and stress transformation matrices derived from dynamic multidegree-of-freedom response computations.⁶ These multiaxial stress components are combined into a uniaxial stress through the Mises criterion of equation (3).

The deterministic method would reduce the tolerance-limit stress components to single values resulting into a worst-on-worst uniaxial applied stress. To derive the statistical tolerance-limit variables from the Mises criterion as required by the reliability criterion of equation (9), the combined mean, standard deviation, and probability range factor are computed through the error propagation law.⁷ This technique consists of expanding the functional relationship in a multivariable Taylor series around a design point (mean) of a system. Applying these statistically derived combined applied-stress variables into the reliability criterion, the reliability design factor is solved for a specified reliability and it is equated to the maximum allowable applied-stress criterion of equation (10). This sizing process is repeated until the disparity coefficient converges to unity.

To predict the reliability of an existing structure, the actual size is substituted into the Mises criterion and equated to the allowable applied-stress criterion. The statistically derived tolerance-limit variables are processed through the reliability criterion, as in the sizing process above, with the disparity coefficient set to unity from which the reliability is solved.

DESIGN UNCERTAINTIES

Neglect of significant design uncertainties is a common cause of premature verification failures. Incorrect assumptions, faulty software, and other errors that can be checked and corrected should not be categorized as uncertainties. Errors that are frequently ignored and that most often degrade the integrity of a built and tested structure are modeling uncertainties. Four basic types are loads, stress, metallurgical, and manufacturing. The latter three modeling uncertainties are assimilated by the real test-article response and, therefore, counteracting margins must be estimated and appropriately implemented in the design analyses.

Reliability response is biased by uncertainties in the two normal distribution variables producing different reliability sensitivities. Those that bias the statistical mean must be included as an accumulated error,

$$e = e_1 + e_2 + e_3 + \dots + e_n.$$

Computational approximations and over-simplified boundary assumptions may be sources of biasing the design mean of the applied stress and must be compensated with the cumulative uncertainty factor acting on the stress mean.

Modeling tolerance uncertainties that are statistically characterized variables and are mutually exclusive may be defined as a multivariable function by combining their dispersions through the following error propagation laws.⁸ When two or more independent variables are added, their standard deviations are root-sum-squared (rss) by the summation function rule, for

$$z = x + y, \quad \sigma_z = \sqrt{\sigma_x^2 + \sigma_y^2}.$$

When independent variables are multiplied and/or divided, their coefficients of variation are rss according to the power function rule, for

$$z = x^n y^m, \quad \eta_z = \sqrt{n^2 \eta_x^2 + m^2 \eta_y^2},$$

where η_z represents the uncertainty coefficient of variation. Structures designed with mean properties and having uncertainty dispersions around the design mean are in this class of error propagation. Manufacturing and material tolerances are examples of the summation function rule. Elastic modulus and Poisson's ratio are defined by multivariables having measured dispersions and should be combined by the power-function rule.

Neglect of variance error would reduce the combined coefficient of variation derived from applied-stress response analysis to

$$\eta_{Ae} = 2\eta_A - [\eta_A^2 + \eta_e^2]^{0.5}, \quad (11a)$$

resulting in a submarginal structure. The variance propagation uncertainties of equation (11a) may be compensated by adjusting the applied-stress tolerance-limit factor of equation (6),

$$B_e = (1 + N \eta_{Ae}) . \quad (11b)$$

Combining propagation errors with applied-stress dispersions and compensating for cumulative errors into equation (9), the specified safety index is satisfied by the reliability design criterion,

$$Z = \frac{(\varphi_D SF) B_e^{-A} + A B_e^{((\varphi_D SF) - e - 1)}}{[\eta_R^2 (\varphi_D SF)^2 B_e^2 + \eta_{Ae}^2 A^2]^{\frac{1}{2}}} . \quad (12)$$

Solving for the reliability design factor with the coupled design disparity coefficient, $\varphi_D SF$, for the specified reliability, the structure sizing proceeds as discussed before.

VERIFIABLE SAFETY INDEX

The first-order reliability criterion response of equation (12) may be verified through the reliability design factor. Since this design factor incorporates the conventional deterministic safety factor defined by equation (1), current static test techniques used to verify deterministic response should also be applicable and available to concurrently verify the reliability criterion response. The verification criterion is thus established by substituting the test derived safety factor SF_T into equation (12) and calculating the test derived safety index response,

$$Z = \frac{(\varphi_D SF) B_e^{-A} + A B_e^{((\varphi_D SF) - e - 1)}}{[\eta_R^2 (\varphi_D SF)^2 B_e^2 + \eta_{Ae}^2 A^2]^{\frac{1}{2}}} . \quad (13)$$

Recognizing that the test applied stress is a predicted operational stress and can only be verified downstream by limited field and flight tests, applying the test verified safety factor to the effective probable range prediction of equation (4) provides another index for structural design acceptance,

$$N_{\text{eff}} = \varphi_D SF_T \left(\frac{1}{\eta_A} + N_A \right) - \frac{1}{\eta_A} . \quad (14)$$

The safety factor probability contribution of the total predicted applied stress is verified by

$$N_T = N_{\text{eff}} - N_A . \quad (15)$$

RELIABILITY SELECTION CRITERIA

Selection criteria concepts being considered range from an arbitrarily agreed on standard value, as fashioned by the deterministic safety factor, to criteria supporting risk analyses. In the absences of an established reliability selection criterion for semistatic structures, it would be interesting to examine briefly the interaction of two concepts with the proposed first-order reliability method.

An immediate demand for a simple and user-friendly reliability selection criterion is to develop a standard reliability derived from the reliability criterion of equation (9). The reliability criterion should be based on a range of design variables representative of successful deterministic design and operational experiences. This approach would not only provide a safety factor versus safety index familiarity, and correlation, but it would also promote designer confidence in the transition. A first-cut design reliability criterion of equation (9) was bounded with a small sample of design variables associated with a current aerospace structure. The resulting minimum reliability exceeded a value of four-nines on operational stress limit (yield stress). It was noted to be most sensitive to the reliability design factor, and an order of magnitude less sensitive to other design variables. The motive for designing to an arbitrarily selected reliability over the arbitrarily selected safety factor, is for designing to a uniform reliability, improved safety audits, and other reliability benefits discussed previously.

One approach to supporting risk analyses is to calculate the risk cost from the currently considered product of probability of failure,

$$p = (1-R), \quad (16)$$

and the cost consequence of that structural failure. Consequence may include cost of life and property loss, cost of operational and experiment delays, cost of inventories, etc. A criterion to balance the risk cost is to equate some proportion of the risk cost with the initial and recurring costs required to provide the structural reliability not to exceed the risk cost. Initial costs would consider the increased structural sizing to the same reliability used in the risk equation through the failure probability of equation (16). Recurring costs include increased propellant, and the increase to payload performance costs caused by the increased structural sizing and propellant weights to accommodate the risk side of the equation.

It would seem that a structural reliability design method is essential for the development of a reliability selection criterion. Since different failure modes may require different reliability design methods, reliability selection criteria should be expected to be failure mode related.

CONCLUSIONS

The conventional deterministic method is the most expedient and dominantly practiced concept for sizing structures. However, it is shown to be genetically flawed and that designing to a specified constant safety factor does not produce uniformly reliable structures. It is incapable of supporting risk analyses, and stress audits of critical structures based on safety factor margins alone do not necessarily identify the weakest regions.

Measured structural data are universally normalized to be applicable to normal probability distribution techniques leading to the simplest reliability design method. The currently developed deterministic design variables and tolerance limits are superimposed on the first-order reliability method to surmount prevailing deterministic deficiencies and to share reliability benefits.

Unlike the arbitrarily selected safety factor of the deterministic method, the equivalent reliability design factor is derived from the reliability criterion for solving and verifying a wide range of practical reliability problems.⁹ The reliability design factor may also be used in the unexpected role of optimizing the structural size to improve payload performance.

The proposed first-order reliability method supports risk analyses and reliability selection criterion development. It may supplement prevailing safety margin audits, and it is suited for probing critical stress regions. It provides uniform reliability and optimum performance structures in support of affordable access to space.

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